THE DEVELOPMENT OF THE INSTRUCTIONAL VISION OF EARLY CAREER SECONDARY MATHEMATICS TEACHERS

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Using narrative inquiry, I share the beginning stories of the instructional visions of secondary mathematics teachers throughout their undergraduate studies of pedagogy and content-specific methods and evidenced in undergraduate coursework and student teaching experience. Guided by Munter's (2014) Visions of High-Quality Mathematics Instruction Role of the Teacher rubric, I answered the following research question: How do the instructional visions of secondary mathematics teachers develop throughout their undergraduate coursework and student teaching? Understanding the instructional visions of mathematics teachers as evidenced in undergraduate coursework and student teaching reveals the impact of discussions, activities, and readings in their education studies and field experiences.

Keywords: Instructional vision; Instructional activities and practices; Preservice teacher education

The work of teachers is like that of orchestra conductors using educational imagination and rules of thumb to continue conversation throughout the class, unlike that of technicians with a set of scientific prescriptions (Eisner, 1983). While conducting their orchestra, teachers make decisions about their teaching and student learning while continually working toward making music. To improve their practice, teachers make several conscious decisions (Leatham, 2006) while working toward *good* or *quality* teaching (Arbaugh et al., 2021; Beswick, 2007; Ernest, 1991; Hammerness, 2001). But what does this encompass? Ernest (1991) initially presented three variations to the definition of *quality* mathematics teaching: (a) students achieve success (e.g., achieving success on standardized assessments), (b) the teacher uses a wide array of teaching strategies, and (c) the teacher is influenced personal epistemology and mathematics-related beliefs system and its realization and transfer into practice. However, only in the third conceptualization does the teacher have much control. In the first two conceptions, there are several conditions outside the control of the teacher (e.g., changing standardized assessments, strategic compliance on practices used in the classroom). Thus, many mathematical researchers (e.g., Ball, 1990; Pajares, 1992; van Putten et al., 2014) focused on identifying teachers' beliefs and the impact of teachers' beliefs on mathematics instruction and students' learning.

Over time, as more researchers continued to study personal (i.e., teacher) impacts on teaching, some researchers (e.g., Arbaugh et al., 2021; Hammerness, 2001; Munter & Wilhelm, 2021) shifted to discussing teachers' vision, or more specifically instructional vision. This shift is a result of the lack of a clear definition of beliefs (Philipp, 2007), since some researchers (e.g., Pajares, 2014; van Putten et al., 2014) have used the term belief without defining it. For example, Parjares (1992) called defining beliefs "at best a game of player's choice" (p. 309). Munter (2014) addressed his use of the term "vision" instead of beliefs as "beliefs suggest a relatively static set of decontextualized ontological commitments" (p. 587), where vision is intended to communicate "a more dynamic view of the future" (p. 587). So while instructional vision and beliefs could be interpreted similarly, the focus of my research will be on the instructional

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visions of secondary mathematics teachers, including its dynamic view of the future, particularly when observing early career teachers striving to put their instructional vision into practice.

Vision has been consistently defined in the literature on mathematics education. Hammerness (2001) defined *teacher vision* as “a set of images of ideal classroom practice for which teachers' strive” (p. 143). Building on Hammerness's (2001) definition, Jansen et al. (2020) defined a *teacher's vision* as “an idealized image of classroom practice” (p. 184) encompassing “their aspirations or hopes of what could occur in their classrooms, schools, communities, and even society” (p. 184). In time, researchers started using more specific phrases, particularly *instructional vision*. Arbaugh et al. (2021) defined *instructional vision* as “a set of internal images that portray the work of teaching mathematics” (p. 449). Similarly, Munter and Wilhelm (2021) defined *instructional vision* “as the discourse that teachers or others currently employ to characterize the kind of ‘ideal classroom practice’ to which they aspire but have not yet necessarily mastered” (p. 343). These discourses include “both descriptions of what high-quality instruction looks and sounds like as well as rationales in terms of what it accomplishes” (p. 343). The instructional vision of teachers, particularly early career teachers, is significant as it depicts what teachers want to see and hear in their classroom.

Teachers' instructional visions matter (Feiman-Nemser, 2001; Viholainen et al., 2014; Woods & Wilhelm, 2020). Looking at the first stage of Feiman-Nemser's (2001) *Central Tasks of Learning to Teach*, learning to teach starts as preservice teachers critically contemplate their beliefs and how they are situated or related to their own vision of good teaching. Sfard (2007), in relation to students' mathematical understanding, found that a change in our discussion around how students develop mathematical understanding needs to happen before students can experience a change in something. The same can be thought of for good teaching; first, teachers discuss the changes they want to see, then they make them happen. Thus, adopting a reflective practice in teaching can lead to changes, often for the better.

Jansen et al. (2020) investigated the impact of teacher education programs on the instructional visions of mathematics teachers. Through online questionnaires, their data consisted of participants self-reported instructional vision for teaching elementary mathematics, the vision promoted by their teacher education program, the influence or lack of influence from their education program on their instructional vision, and if they thought their visions changed over time. Their findings raised awareness around how teacher education programs can impact the instructional visions of their graduates as they found an alignment related to developing students' conceptual understanding and engaging students in a productive struggle in their teachers' instructional visions and pre-service program experiences. However, they found these novice instructional visions can change with more classroom experience and professional development.

Arbaugh et al. (2021) and Munter and Wilhelm (2021) extended the body of research on the instructional vision of teachers. Arbaugh et al. (2021) examined, in a semester-long course, changes in the instructional visions of secondary mathematics preservice teachers of teaching mathematics. Although they found a positive change (e.g., moving from understanding teaching as telling to teaching as facilitating), they wondered about the permanence of such changes. This identified the need for longitudinal studies “to examine whether such changes in vision persist into teachers' early careers, including whether... [they] continue to change, stagnate, or revert” (p. 459). Munter and Wilhelm (2021), through the lens of “how teachers develop more sophisticated instructional visions” (p. 344), investigated instructional vision of middle school mathematics teachers. They found several factors contributing to the development of a more sophisticated instructional vision, including: (a) mathematical knowledge for teaching, (b)

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instructional practice, (c) instructional views of colleagues, and (d) frequency of interaction with colleagues. Jansen et al. (2020), Arbaugh et al. (2021), and Munter and Wilhelm (2021) encouraged looking to teachers' environments (e.g., school and community) when making sense of their instructional vision and any changes that might or might not be occurring.

My Positionality

As a former high school mathematics teacher, Sfard's (2007) notion of first changing how we discuss mathematical learning of students before changes take place resonated with me as I reflected on my own ideas before acting on them. I recall thinking about my future classroom as an undergraduate. Many professors challenged me in various courses to think about what we wanted our classroom to be like, what students would be doing, and what we, the teacher, would be doing. Woods and Wilhelm (2020) similarly noted the importance of early career teachers' instructional visions and how their initial instructional vision can impact the pedagogical resources they use. They also claimed that while teachers' instructional visions are often evolving, their early career instructional visions are often the most critical as early career teachers begin to implement various instructional practices. This resonated with me in reflecting on my own experience in the classroom. Just as I have had my own unique experiences related to the evolution of my instructional vision throughout nine years in a secondary mathematics classroom, so have other teachers. Thus, in my longitudinal research study, I examine the instructional visions of secondary mathematics teachers throughout their early career. Here, I share the initial phases of study (i.e., undergraduate coursework and student teaching) guided by my research question, *How do the instructional visions of secondary mathematics teachers develop throughout their undergraduate coursework and student teaching?*

Theoretical Framework

In analyzing secondary mathematics teachers' instructional visions, I used Munter's (2014) Vision of High-quality Mathematics Instruction (VHQMI) Role of the Teacher rubric. Munter's VHQMI rubrics focus on the development of teachers' visions of high-quality mathematics instruction. The development of teachers' visions is one key that differentiates Munter's definition and rubrics from others. Munter's VHQMI rubrics were developed from other theoretical perspectives, including Carpenter and Lehrer's (1999) notion of *learning with understanding*, Cobb and Bauersfeld's (1995) *emergent perspective* on learning, and Rogoff et al.'s (1996) *community of learners*. Through conducting research in their own classrooms, Carpenter and Lehrer (1999) found five forms of classroom activity through which mathematical understanding can emerge: "(a) constructing relationships between current and more sophisticated ways of thinking; (b) building rich, integrated knowledge structures; (c) reflecting; (d) articulating what one knows; and (e) making mathematical knowledge one's own by coming to perceive it as evolving and authoring one's own learning" (Munter, 2014, p. 589). Carpenter and Lehrer's (1999) five forms of activity align with Cobb and Bauersfeld's (1995) emergent perspective, as the five forms of activity are social practices through which individual knowledge construction, as well as mathematical learning, occur. Engaging in these social practices, we have the teacher and the students, or the community of learners as defined by Rogoff et al. (1996), existing somewhere on a continuum between teacher-led and student-led.

These combined perspectives contributed to Munter's (2014) VHQMI, which he divided into three dimensions: role of the teacher, classroom discourse, and mathematical tasks. In this study,

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I focus on Munter's (2014) role of the teacher: (a) *motivator*, (b) *deliverer of knowledge*, (c) *monitor*, (d) *facilitator of knowledge*, and (e) *more knowledgeable other* (see Table 1).

Table 1: Abbreviated VHQMI Role of the Teacher (Woods & Wilhelm, 2020, p. 117)

Role of the Teacher	Description
Motivator	Describes the teacher as being able to captivate and hold students' attention.
Deliverer of knowledge	Describes the teacher as the primary source of knowledge. Here, the teacher focuses on mathematical correctness and his/her thoroughness of explanations.
Monitor	Describes the teacher as the primary source of knowledge. However, the teacher emphasizes the importance of providing time for students to work together to make sense of what he/she has demonstrated.
Facilitator of knowledge	The teacher is focused on "reform instruction" but may not have a strong understanding of the pedagogical strategies that underlie those forms.
More knowledgeable other	The teacher describes his/her role as proactively supporting students' learning through participation. Further, the teacher emphasizes the importance of designing a learning environment that supports problematizing mathematical ideas, giving students voice and mathematical authority, and holding students accountable for their learning.

Munter (2014) used other existing research on effective mathematics instruction (i.e., Hiebert et al., 1997; Jackson et al., 2013; Lampert, 1990; Lobato, Clarke, & Ellis, 2005; Madsen-Nason & Lappan, 1987; Staples, 2007; Stigler & Hiebert, 1999) and data collected as part of the Middle School Mathematics and the Institutional Setting of Teaching (MIST; Cobb & Smith 2008) to identify three ways in which to characterize the role of the teacher: "(a) conception of typical activity structure (i.e., the teacher's general mode of instruction and role in classroom activities), (b) attribution of mathematical authority (i.e., students' roles with respect to the mathematics being learned), and (c) influence on classroom discourse" (p. 600). These three characterizations were then factored into the descriptions of the roles of the teacher.

First, Munter (2014) described a teacher as a *motivator*, or one who captivates and maintains the students' attention. For teachers who are *deliverers of knowledge*, instruction primarily encompasses teacher-to-student discourse with the teacher providing direct instruction with little discussion. The validity of mathematical tasks comes from the textbook or the teacher (Simon, 1994) when teachers are the *deliverers of knowledge*. Teachers who play the role of *monitors* supervise student-to-student discussion. If students stray toward inaccurate reasoning, the teacher resets them (Munter, 2014). According to Stigler and Hiebert (1999), the activity structure when teachers are *monitors* encompasses a two-phase *acquisition and application* lesson. The teacher leads a discussion or demonstration, and then the students work in groups with what was just demonstrated, while the teacher circulates around the classroom.

When looking at teachers who are the *facilitator of knowledge* and *more knowledgeable other*, the role of the teacher shifts as instruction becomes more student-led (Munter, 2014). Lobato, Clarke, and Ellis (2005) describe the classroom discourse with a *facilitator of knowledge* as including student-to-student talk with the teacher facilitating and hesitant to guide students so

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as not to interfere with the discovery process. Students are discovering and teaching themselves. Unlike when the teacher is a *monitor*, if students begin to stray toward inaccurate knowledge, the teacher attempts to guide students back on track through questioning (Munter, 2014). When teachers *facilitate knowledge*, Madsen-Nason and Lappan's (1987) *launch-explore-summarize* (LES) structure of a lesson unfolds with the teacher posing a task(s), the students working, typically in groups, to complete the task(s), and the students revealing and explaining their solutions at the end of the lesson. The teacher may also clarify the concept if necessary.

Teachers as *more knowledgeable others* go beyond facilitating (Munter, 2014), while continuing to model Madsen-Nason and Lappan's (1987) *launch-explore-summarize* lesson structure. Students' ideas are elicited to create a shared context through discussions (Staples, 2007). The teacher supports the learning of the students. Students work toward a shared goal through problematizing content (Hiebert et al., 1997), and instead of validity being provided by the textbook or teacher, it comes from the classroom community (Simon, 1994) as the students and teacher share the authority in learning (Lampert, 1990).

Methods

This project is currently in Phase III of a longitudinal case study (Merriam & Tisdell, 2016) in which I am investigating the instructional visions of six secondary mathematics teachers from their undergraduate studies into their early careers. The findings shared here will include data from both Phase I, undergraduate coursework, and Phase II, student teaching as data in Phase III, interviews during their first year of teach, is still ongoing in regard to data collection and analysis. To gain an understanding of each early career secondary mathematics teacher, I will compile an in-depth description and analysis in their respective contexts will be conducted as part of the case study (Merriam & Tisdell, 2016; Yin, 2014).

Participants, Context, and Data Sources

The six participants, identified using pseudonyms, completed their undergraduate studies in secondary mathematics education at a large midwestern university. The participants included three self-identifying males and three self-identifying females. One of the female participants is African American, while the other five participants are Caucasian. Two participants, one male and one female, completed their student teaching in a midwestern rural school, while the others completed their student teaching in midwestern urban schools. In addition to student teaching, each participant spent at least 30 hours in other middle school and high school mathematics classrooms and also individually taught a section of an entry-level college algebra course at their large midwestern university during the semester prior to student teaching.

The secondary mathematics education program in which the participants were enrolled was focused on collaborative, student-centered learning designed to develop students' conceptual understanding, mathematical reasoning, and problem solving in addition to their procedural fluency. Students wrote several lesson plans following the LES model of Madsen-Nason and Lappan (1987) and were asked to consider knowledge about their students that influences their planning, a universal design for learning with multiple learning supports through representation, action, expression, and engagement (Hunt, 2011), as well as an assessment plan (both summative and formative), differentiation, and accommodations. All of these elements of lesson planning, including how to successfully lead and launch, and what makes a good mathematical task, were discussed extensively in their two secondary mathematics methods courses. Students practiced peer teaching before teaching in their field experiences in conjunction with the methods courses.

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Three sources of written data were collected during Phase I: (1) philosophy of education originally drafted in an introductory education course (Artifact I), (2) mathematics classroom vision written a year before student teaching in the first of two secondary mathematics methods courses (Artifact II), and (3) revised philosophy of education written the semester prior to student teaching in the second of two secondary mathematics methods courses (Artifact III).

The data collected in Phase II included two semi-structured interviews (Merriam & Tisdell, 2016) during student teaching, one about half-way through their experience (Interview IA) and another one at the conclusion of their student teaching (Interview IB). Additional data sources for the longitudinal study will include two semi-structured conversational interviews during each of their first two years of teaching mathematics (Phase III and Phase IV). Additionally, another source of data takes the form of research memos I continue to write throughout the data collection and analysis my study (Bogdan & Biklen, 2011).

Data Analysis: Interpreting Instructional Visions

As I began data analysis, I read, re-read, and took notes on key elements in the artifacts to make meaning of what was represented both implicitly and explicitly in the data (Bogdan & Biklen, 2011; Flick, 2014). In reading and re-reading the artifacts, I looked for themes around Munter's (2014) descriptors of the role of teacher, including student talk, launch, explore, summary, groups, and conversations related to the description of the discourse in the classroom, the structure of the activity, and who had the mathematical authority in the classroom. Triangulation amongst each of the artifacts and research memos was used to check the validity and reliability (Merriam & Tisdell, 2016). These key words and descriptors combined with my initial notes allowed me to determine how each of the instructional visions of the participants aligned with Munter's (2014) VHQMI Role of the Teacher rubric.

After collecting all the interviews during student teaching and transcribing them, I began the same process in looking for the same key descriptors. I completed this process for each participant before moving on to the interview transcripts of the next participant. Throughout all of the analysis I looked for reoccurring regularities or shifts in the key descriptors (Merriam & Tisdell, 2016) to uncover the alignment with Munter's (2014) VHQMI Role of the Teacher rubric.

Findings

Here, I share the development of the instructional vision of Allison and Corey as they represent the extremes in my sample. Allison's instructional vision was challenged in her students teaching, while Corey's instructional vision was refined. The obstacles Allison faced while further developing her instructional vision were unlike others, as was the freedom Corey experienced in trying new ideas while cultivating his own instructional vision. I focus on the key descriptors found in their undergraduate coursework (Artifact I, II and III) and the transcripts of the semi-structured interviews during their student teaching experience (Interview IA and IB) and how they correspond to Munter's (2014) role of the teacher rubric.

Allison

Allison, a Caucasian female, described her classroom as having students in clusters so that students could "communicate with each other" (Artifact I). In her philosophy (Artifact II), she described how "teachers can best help students learn by making them feel supported, rather than being dominated by an authority figure. Teachers should guide students toward success rather than forcing end results upon them." Similarly, she noted: "if the students have some input on how the class should run, they will be more likely to follow the rules and engage in the class

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because they will feel autonomous." These key descriptors focused on the active role students play in the mathematics classroom and Allison's role as the *more knowledgeable other*. Another key descriptor found in her philosophy of education was how she planned to "teach math using LES (launch-explore-summarize) and not focusing on lecturing for students to just understand the concepts surface-level and procedurally." This indicates her role as *a facilitator of knowledge* or a *more knowledgeable other* depending on the specific LES lesson.

Allison, who attended a large urban midwestern high school, completed her 16 weeks of student teaching at a small rural midwestern high school. She described in her first interview her cooperating teacher as almost completely opposite in regard to instructional vision, as she was a *motivator* or *deliverer of knowledge*. She found challenges with adjusting to this environment where many students do not graduate from high school and her cooperating teacher's style.

About a third of the way into her student teaching, Allison received new groups of students when the third trimester started in early March. Allison started teaching full-time and was hopeful that the student-centered strategies she had previously attempted would be more successful with new groups of students. Allison wanted to try the LES structure and wanted her students to work collaboratively. But while Allison's instructional vision still appeared to align with the characteristics of *facilitator of knowledge* and *more knowledgeable other*, her classroom became more of a place in which she was the *monitor* or *deliverer of knowledge*, the place at which her students were most comfortable.

Allison encountered additional challenges in all three of her classes (Algebra I Repeat, Algebra II, and Finite) where students did not have the requisite prior knowledge to learn the new content. For example, in Interview IB, Allison talked about her three-day plan for teaching piecewise functions in Algebra II. On the first day, the plan "exploded" as the students did not have a conceptual understanding of how to find the domain and range of functions on a graph. As a result, Allison shifted to direct instruction to help her students understand domain and range. Allison noted she wanted to "improve what I am doing,..., and to not have to teach to the test. I hate that. I like want the students to understand things more than just the problem that is on the test" (Interview IA). So, while Allison's student teaching turned into primarily direct instruction based on her circumstances and the role she exhibited was a *motivator* and a *deliverer of knowledge*, she was not happy and was eager to try something different in her own classroom. Following Allison into her first year of teaching will allow me to see how her instructional vision continues to develop.

Corey

"Accessibility and group dynamics will be very important to the physical layout," Corey, a Caucasian male, noted in his classroom vision (Artifact I). Corey's "goal is to lower as many physical barriers as possible in the way of students' conversations and ideations." These descriptors demonstrate Corey's eagerness for student participation and student-centered learning as the *facilitator of knowledge* or the *more knowledgeable other*. Corey continued to discuss student engagement by noting the importance of questions and how "students in my class should know that their ideas and questions are valuable." Corey envisioned his students discovering math by questioning and seeking their own answers. Corey went on to state, "rarely do I want to just tell students what the correct answer or correct method is; I'd like to encourage them to look inward to see if they can't follow their own intuition to a solution."

Corey's revised philosophy of education (Artifact II) echoed much of his previous classroom vision. In his philosophy of education, he added elements related to student learning, including productive struggle and "that (productive) struggle is necessary to conceptual understanding." He

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compared solving real world problems to that of productive struggle in a mathematics classroom: "very few problems in the real world come with pre-defined solutions, and most of them require some critical thinking, trial and error, and logical reasoning skills to solve." He concluded, "I still believe that my classroom can be a valuable asset to students' development as critical thinkers and persistent learners." These descriptions in his philosophy (i.e., questioning, student thinking and discovery, productive struggle, trial and error) situate Corey's instructional vision as that of a *facilitator of knowledge* or a *more knowledgeable other*.

Corey went on to student teach at a small primarily Caucasian rural midwestern junior/senior high school and worked with three cooperating teachers while teaching two sections of seventh grade math, a section of co-taught Algebra I with freshman, a section of Algebra I with eighth graders, and two sections of trigonometry with eleventh and twelfth graders. In working with three cooperating teachers and teaching three different courses, Corey reported learning a lot as he continued to refine his own instructional vision while trying to put some of his ideas into practice. In Interview IA, Corey mentioned was how he was finally starting to build student-teacher relationships with his students, and more so 7^{th} graders. He found that his lessons were improving because he was able to relate to his students more. When talking about his lessons, Corey often talked about the launch, explore, and summary, providing evidence for his use of the LES model and situating himself as the *facilitator of knowledge* or the *more knowledgeable other*. In Interview IB, Corey described how his students would offer ideas and suggestions, but noted that he was leading them or acting as the *facilitator of knowledge*. Corey's variety of classes and cooperating teachers facilitated his exploration of activities and teaching styles, allowing him to take the role of *facilitator of knowledge* or even a *more knowledgeable other*.

Discussion and Implications

Perhaps not surprisingly, as Jansen et al. (2020) and Arbaugh et al. (2021) found similar results, the instructional visions of the secondary mathematics teachers were similar to and aligned with the ideals emphasized in their secondary mathematics education program, particularly relative to their undergraduate coursework (i.e., classroom vision and philosophy of education). The key words and phrases in their writing and in the interview transcripts aligned closely with the ideals of Munter's (2014) VHQIM role of teacher as *facilitator of knowledge* and the *more knowledgeable other.* Their undergraduate coursework emphasized the LES model, which was often mentioned in the interviews and in their descriptions of ideal lessons. In addition, collaborative learning through group work and class discussions with student talk was emphasized in their writing and interviews. Knowing the presence of these similarities between the instructional visions of secondary mathematics teachers and the approach of the secondary mathematics education program the teachers attended shows the impact of one's program on individual teachers. This stresses the importance for us as mathematics teacher educators to share with our future mathematics teachers how mathematics can be taught even though it may be different from how one learned mathematics or even observes mathematics being taught in the field. Readings, reflections, activities, and discussions in the undergraduate coursework impact the instructional visions of secondary mathematics teachers. The instructional vision of the cooperating teachers and classroom structures while student teaching also shaped their instructional visions. For Allison, she was challenged by this and experienced teaching in ways she herself was not happy with, but worked in the environment in which she was in. Corey, on the other hand, with multiple cooperating teachers, was able to try more while continuing to build on his instructional vision that he developed in his undergraduate coursework. As Arbaugh

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and colleagues (2021) questioned the permanence of the instructional visions of early-career mathematics teachers, there is still much to be explored as my study continues to follow these teachers into their early career in their own classrooms. One area of future exploration could be to include the instructional vision of the novice teachers' mentor teachers in order to see the possible impact of their cooperating teacher more clearly. Through continuing my study, I hope to see how these early career teachers begin to conduct their own orchestras (Eisner, 1983).

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